# FROM CUSPS TO CORES: A STOCHASTIC MODEL

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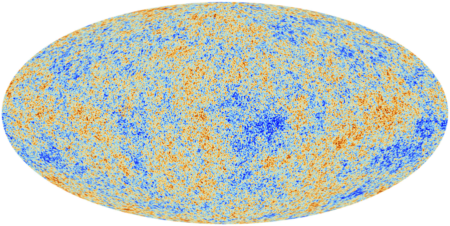
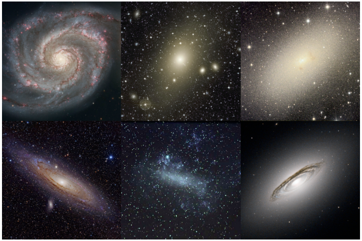
# Galaxy formation in the ΛCDM model

From a very homogeneous early Universe to the current distribution of galaxies, clusters and voids...





- **The standard ΛCDM cosmological model:**
  - 26% cold dark matter (CDM)
  - 5% baryons (ordinary matter)
  - 69% dark energy (accelerated expansion, Λ)
- The Universe is initially very homogeneous (cf. cosmic microwave background, 380 000 years after the Big Bang).
- **Gravitational attraction** vs. the expansion of the Universe.
- **Hierarchical dark matter dynamics, baryons cool and contract within dark matter haloes.**

Images: ESA/Planck collaboration/C. Mihos/ESO/A. Block/NOAO/AURA/NSF/ A. Evans/NASA/S. Beckwith/Hubble Heritage Team/STScI/AURA/Skatebiker

Volker Springel et al. (2008)

# Star formation & feedback processes

Structure formation is mostly driven by dark matter dynamics at cosmological scales, but baryonic processes become important at galactic scales.

## Star formation

## Stellar feedback mechanisms

- Strong radiation fields
  - UV ionizing radiation from young stars: heats the gas up to $10^4$ K and photodissociates $H_2$
  - Photoevaporation (& rocket effect)
  - Radiation pressure
- Stellar winds
- Supernovae explosions

## Active galactic nuclei (AGN) feedback

- Radiation
- Outflowing winds
- Highly-collimated jets

## Positive feedback:

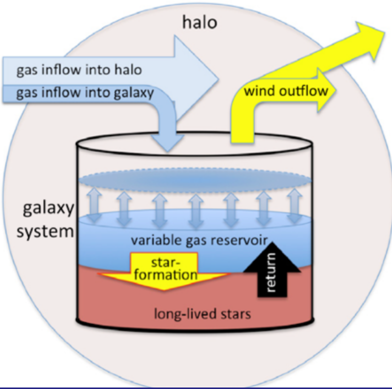
- heavy elements enhance cooling
- compression waves

Vogelsberger et al. (2014)

# Gas and stellar cycles within galaxies

A galaxy can be seen as a gas reservoir fed by accretion and emptied through star formation and outflows. Some of the outflowing gas can fall back onto the galaxy and be recycled (bathtub model: Bouché et al. 2010, Dekel & Mandelker 2014).

$$\underbrace{\dot{M}_{gas}}_{\text{gas reservoir}} = \underbrace{\dot{M}_{gas,in}}_{\text{inflows}} - \underbrace{\dot{M}_{stars}}_{\text{star formation}} - \underbrace{\eta \dot{M}_{stars}}_{\text{outflows}} + \underbrace{R \dot{M}_{stars}}_{\text{recycling}}$$




Lilly et al. (2013)

# Challenges of the ΛCDM paradigm at galactic scales

- **“Downsizing”**

  Massive galaxies host the oldest stellar populations, which seems to contradict the hierarchical ΛCDM model. But star formation is more efficient for haloes of mass between $10^{10} - 10^{12}$ $\mathrm{M}_{\odot}$ (cf. galaxy luminosity function).

- **The “angular momentum catastrophe”**

  Early simulations with minimal feedback models formed disks that were too small as the gas cooled excessively and transfered its angular momentum to the dark matter. Stronger feedback models yield more realistic disks.

- **Bulgeless giant galaxies**

  The ΛCDM model predict bulges (from mergers or clump evolution within the disk), while many bulgeless giant galaxies are observed in the vicinity of the Milky Way.

- **The “too big to fail’ problem’**

  Numerical simulations predict too many subhaloes than observed for galaxies like our Milky Way, and the simulated subhaloes are too big to fail to form stars so we should observe them. Also, the most massive simulated subhaloes don’t seem to match the observed ones in terms of mass (simulated ones too massive).

- **The core-cusp discrepancy**

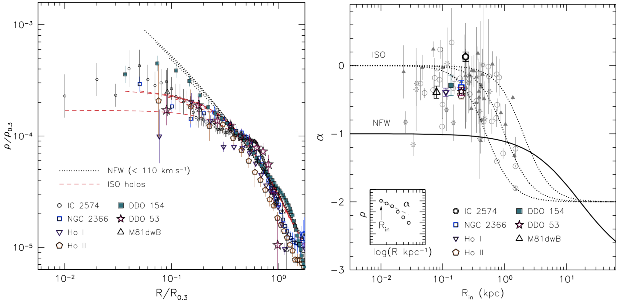
# The core-cusp discrepancy

- Dark matter cosmological simulations predict a universal **'cuspy'** density profile for dark matter haloes, with $\rho \propto r^{-1}$ at the center.

NFW profile (Navarro et al. 1996)

$$\rho(r) = \frac{\rho_0}{\frac{r}{\mathrm{R_s}}\left(1+\frac{r}{\mathrm{R_s}}\right)^2}$$

- Observations of dark matter dominated dwarf, low-surface-brightness and dwarf satellite galaxies instead show that the dark matter distribution is **'cored'**.




Oh et al. (2011): dwarf galaxies from the THINGS

- The core-cusp discrepancy is related to other challenges of the ΛCDM paradigm at galactic scales: in particular, the 'too big to fail', the persistence of galactic bars, and the transfer of angular momentum from the baryons to the dark matter.

# Attempts at solving the problem

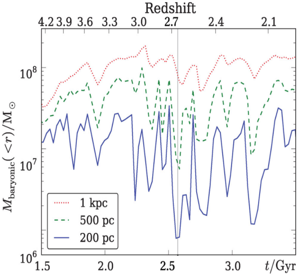
- **Solutions that fundamentally change the physics of the cosmological model:**
  - **warm dark matter**: could in principle yield more diffuse dark matter structures, but the density profiles are often still cuspy & the power spectrum might conflict with the Ly$\alpha$ forest
  - **self-interacting dark matter**: becomes collisional in the inner halo, heats up and produces a shallower density profile; problems at cluster scales
  - **exotic cut-offs in the matter power spectrum**: 'fuzzy' ultra-light particles predicted by string theory; asymmetric, repulsive, or fermionic dark matter; alternative inflation models; etc.
  - **MOND** (Milgrom 1983): fits galactic rotation curves, no cusp-core problem, but lacks an underlying relativistic theory so can't be extended to cosmological problems for the moment; mass discrepancies in clusters, which may still require a form of dark matter
- **Solutions invoking baryonic processes within the $\Lambda$CDM framework:**

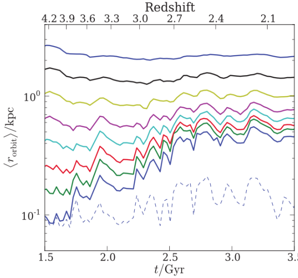
  Baryons play an important role at galactic scales, and can interact gravitationally with the dark matter. In particular, recent simulations with feedback are able to reproduce cored density profiles. What are the precise mechanisms through which baryons can affect the dark matter distribution?

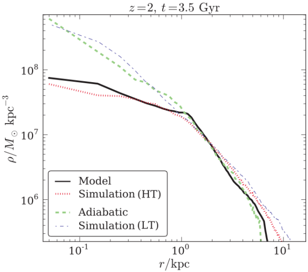
# How can baryons affect the dark matter halo

- **Adiabatic contraction**
  When baryons cool and contract, they accumulate at the center of the halo, which steepens the potential well and causes the dark matter to contract as well.
- **Dynamical friction**
  When a massive object (satellite galaxy, clump of gas) moves with respect to a background of smaller particles, it loses part of its energy as the concentration of particles increases in its wake and generates a drag force. El-Zant et al. (2001, 2004): cores.
- **Repeated potential fluctuations from feedback processes**

Pontzen & Governato (2012): repeated, non-adiabatic potential fluctuations from feedback can heat the dark matter and lead to the formation of a core.



# An analytical model

- To isolate further the physical mechanism through which dark matter gains energy, we present and test a theoretical model in which the gravitational potential fluctuations leading to core formation arise from feedback-induced **stochastic density variations** in the gas distribution.
- **Our goal:**
  - characterize the effects of such variations on the dark matter particles in a statistical sense: resulting velocity variance, relaxation time
  - test the model with numerical simulations, notably in the case of dwarf haloes
- **Assumptions:**
  - an unperturbed homogeneous gaseous medium of density $\rho_0$;
  - isotropic density perturbations within a sphere of radius $d$, i.e., within a volume $V \propto d^3$;
  - a power-law power spectrum

$$\mathcal{P}(k) = V\ \langle|\delta_{\vec{k}}|^2\rangle = VCk^{-n};$$

  - minimum and maximum scale lengths $\lambda_{min}$ and $\lambda_{max}$ (or alternatively, $k_{max} = 2\pi/\lambda_{min}$ and $k_{min} = 2\pi/\lambda_{max}$);
  - a stationary process.

Fourier decomposition of the density contrast $\delta(\vec{r})$ and of the resulting force on the dark matter particles:

$$\delta(\vec{r}) = \frac{V}{(2\pi)^3} \int \delta_{\vec{k}}\ e^{i\vec{k}.\vec{r}}\ d^3\vec{k}$$

$$\phi_{\vec{k}} = -4\pi G\rho_0\ k^{-2}\ \delta_{\vec{k}}$$

$$\vec{F}_{\vec{k}} = 4\pi i\ G\rho_0\ \vec{k}\ k^{-2}\ \delta_{\vec{k}}$$

## Velocity variance

- Integrating the equation of motion of a dark matter particle and averaging over the different particles enables to write the velocity variance after a time $T$ in terms of the force time correlation function:
$$\langle \Delta v^2 \rangle = 2 \int_0^T (T-t) \langle F(0)F(t) \rangle \, dt.$$
- Introducing $v_r$ the characteristic velocity of the dark matter particles with respect to the fluctuating field, $r = v_r t$ and $R = v_r T$ the distances a test dark matter particle travels relatively to the field during time $t$ and $T$ yields
$$\langle \Delta v^2 \rangle = \frac{2}{v_r^2} \int_0^R (R-r) \langle F(0)F(r) \rangle \, dr.$$
- Injecting the expression of the force auto-correlation function
$$\langle F(0)F(\vec{r}) \rangle = \frac{V}{(2\pi)^3} \int |\vec{F}_{\vec{k}}|^2 \, e^{i\vec{k}.\vec{r}} \, d^3\vec{k}$$

and assuming $k_{max} \gg k_{min}$ and $k_{min} R \gg 1$, we obtain with $D = 8CV(G\rho_0)^2$
$$\langle \Delta v^2 \rangle \sim \frac{\pi D}{n v_r} \frac{T}{k_{min}^n}.$$

- The velocity dispersion is set by the largest fluctuation scale $\lambda_{max} = 2\pi / k_{min}$.
- The assumption $R \ll 1/k_{min}$ corresponds to the diffusion limit where the small persistent density fluctuations initiate random walks for the dark matter particles

# Relaxation time

In stellar dynamics, stars get deflected by their 2-body interactions with each other. $\Delta v$ grows progressively and eventually reaches its original velocity $\langle v \rangle$. $t_{\mathrm{relax}}$ is the time at which $\Delta v = v$.

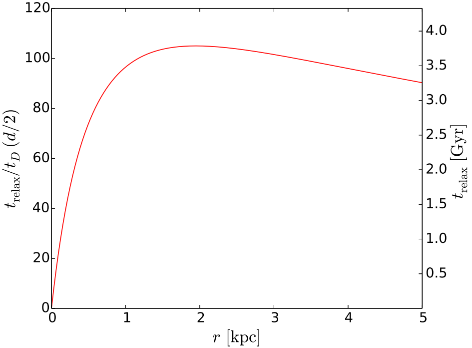
- Here,

$$t_{\mathrm{relax}} = \frac{n v_r \langle v \rangle^2}{8\pi (G\rho_0)^2 V \langle |\delta_{k_{min}}|^2 \rangle}.$$

mean unperturbed orbital velocity at radius $l$: $\langle v \rangle \sim l/t_D(l) \sim l\sqrt{G\rho(l)}$.
$v_r$ due to the largest fluctuation scale: $v_r \sim d\sqrt{G\rho(d)}$.

- Evaluation of $t_{\mathrm{relax}}$ for a fiducial dwarf halo with NFW density profile

$\mathrm{M_{vir}} = 2.26\ 10^{11}\ \mathrm{M_\odot}$
$\mathrm{R_{vir}} = 30\ \mathrm{kpc}$
$\mathrm{R_s} = 0.9\ \mathrm{kpc}$
$d/2 = 5\ \mathrm{kpc}$
$f(d/2) = 17\%$
$n = 2.4$
$\langle \delta^2_{k_{min}} \rangle = 0.005$




The relaxation time gives a time scale at which the density variations are expected to affect the trajectories of the dark matter particles, but does not specify the global response of the system.

# Numerical test setup

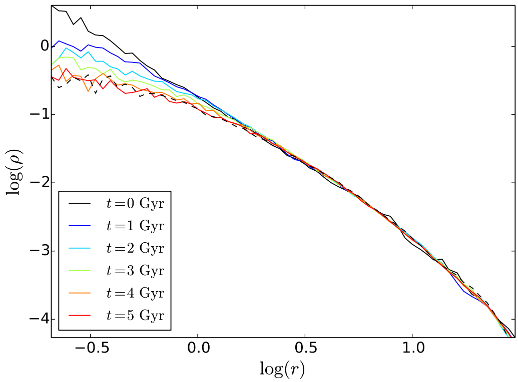
- **The self consistent field (SCF)** method developed by Hernquist & Ostriker (1992):
  - collisionless systems
  - at each time step: computes the gravitational potential and advances the particles' trajectories accordingly
  - density and potential expanded in a set of basis functions deriving from spherical harmonics
  - expansion cut-off radial and angular numbers, $n_{max}$ and $l_{max}$ ($\sim$ softening)
    optimal choice for $\sim 10^5$ particles: $n_{max} = 10$, $l_{max} = 4$ (Vasiliev 2013)

- **Initial conditions:** the fiducial dwarf NFW halo previously defined

- **Density perturbations as in the analytical calculations:**
  - power-law power spectrum
  - $\lambda_{min} = 10\ \mathrm{pc}$, $\lambda_{max} = 1\ \mathrm{kpc}$
  - added in the code through their effect on the force and on the gravitational potential
  - random direction of the force on each particle
  - the pulsation frequency associated to a mode $k$ is defined
    - either as $\omega(k) = v_g k$ with a constant propagation velocity, $v_g = d/t_D(d)$.
    - either as $\omega(k) = 2\sqrt{k}$ from Larson's relation $\sigma \propto k^{-0.5}$
      ($\omega^{-1}$ in units of 10 Myr and $k$ in kpc$^{-1}$)
  - the total force is rescaled a posteriori to match the assumed power-spectrum normalization

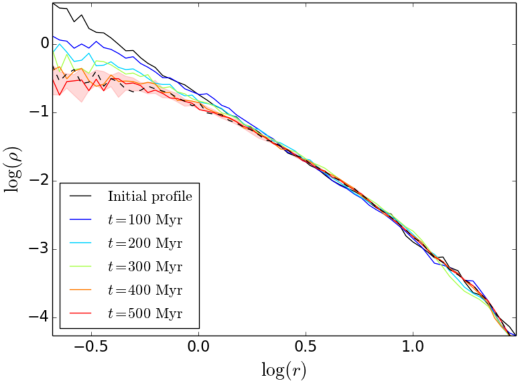
# Simulation results: the spherical case ($l_{max} = 0$)

- The assumed stochastic density fluctuations lead to the formation of a core in an initially cuspy configuration within a timescale comparable to the relaxation time derived analytically: **The fluctuations leading to core formation may be modeled as stochastic processes and their dynamical effects as a diffusion process.**
- The effects mostly depend on the fluctuation level and the gas fraction. As expected from the expression of the relaxation time, the dependence in $n$ is weak. $\lambda_{min}$ and $\lambda_{max}$ do not affect the resulting core (understandable in terms of diffusion limit).



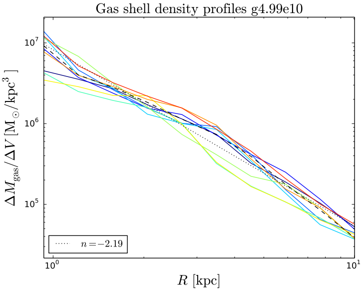
# Simulation results: non-radial modes ($l_{max} \neq 0$)

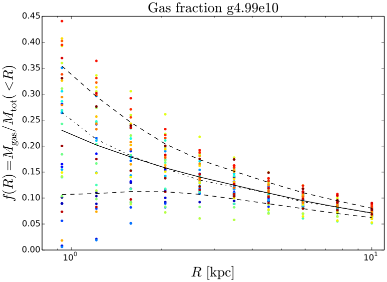
- Removing spherical symmetry leads to an accelerate cusp-core transformation, with the same parametrization.
- The processes through which the energy stemming from the fluctuations is redistributed within the halo could be largely non-isotropic. Asphericity seems to be a key ingredient for an efficient cusp-core transition (cf. also Pontzen et al. 2015).



## Perspectives

- Inputs from hydrodynamical simulations with different implementations of feedback (stellar, AGN, etc.) to specify the statistical properties of the fluctuating density field and better understand the feedback models. Collaborations with Andrea Maccio & Liang Wang (NIHAO project), Justin Read, Andrew Pontzen, James Bullock.
- Generalize the calculations to a non-homogeneous gas distribution, relate our stochastic model with the bathtub model, episodes of inflows/violent outflows, and the calculations of Pontzen & Governato (2012). Collaboration with Avishai Dekel.

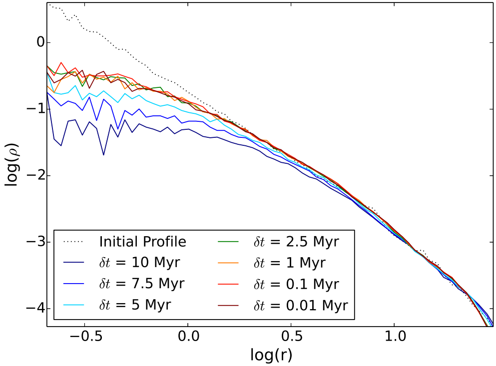



Gas distribution from a NIHAO dwarf galaxy

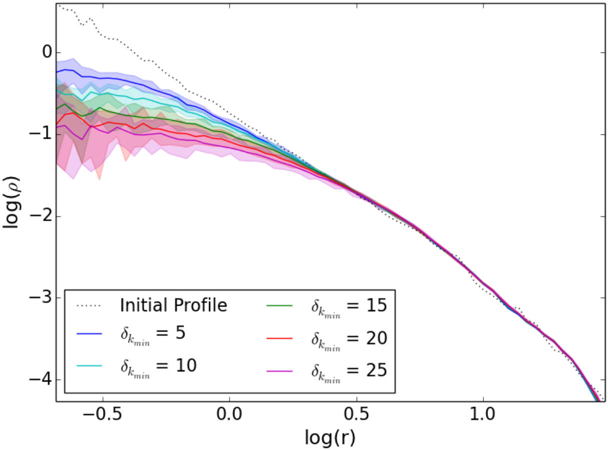
# Influence of the time step

- Time scales associated to the fiducial perturbations, with $\lambda_{min} = 10$ pc and $\lambda_{max} = 1$ kpc.
  - When $\omega(k) = v_g k$ with $v_g = d/t_D(d) = 134\ \mathrm{kms}^{-1}$:
    - $T(k_{min}) = 2\pi/\omega(k_{min}) = 7.5\ \mathrm{Myr}$
    - $T(k_{max}) = 2\pi/\omega(k_{max}) = 0.075\ \mathrm{Myr}$
  - When $\omega(k) = 2\sqrt{k}$:
    - $T(k_{min}) = 12.5\ \mathrm{Myr}$
    - $T(k_{max}) = 1.3\ \mathrm{Myr}$
- As the perturbations are dominated by those near the largest scale $\lambda_{max} = 2\pi/k_{max}$, the simulation already converges for timesteps $\delta t \lesssim 2.5\ \mathrm{Myr}$.
- We carried most of our simulations at $\delta t = 0.1\ \mathrm{Myr}$, which gives similar results as those at $\delta t = 0.01\ \mathrm{Myr}$.



# Influence of the fluctuation level $\delta_{k_{min}}$

- Evolution after 500 Myr for different fluctuation levels averaged over ten realizations and corresponding scatter.



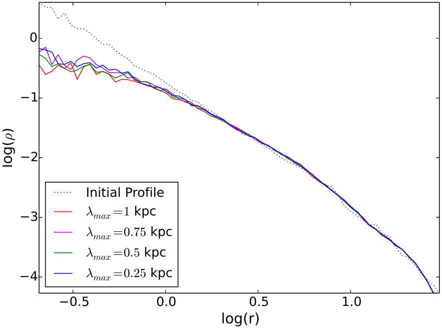
# Influence of $\lambda_{min}$ and $\lambda_{max}$

- The normalization of the force only weakly depends on the minimum fluctuation scale $\lambda_{min} = 2\pi/k_{max}$ as $k_{min} \ll k_{max}$ but does depend on the maximum fluctuation scale determined by $k_{min}$:

$$\langle F(0)^2 \rangle = \frac{8\,(G\rho_0)^2\,\langle|\delta_{k_{min}}|^2\rangle\; d^3}{n-1}\; k_{min}.$$

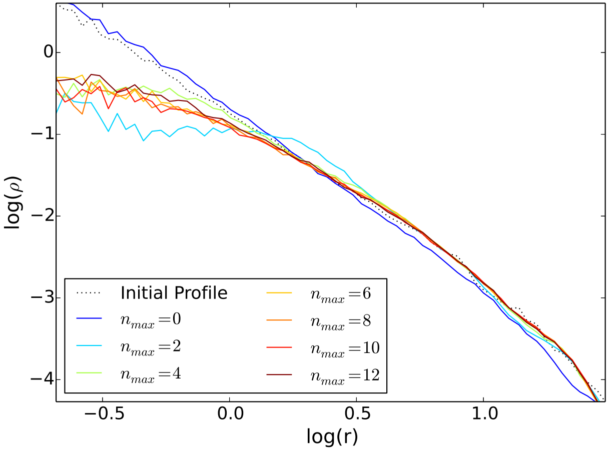
- Accordingly, the effect of the perturbations does not depend on $\lambda_{min}$. But it does not depend on $\lambda_{max}$ either. Why?

- In a diffusion process, particle trajectories are affected by small successive kicks:
  - at each kick, $\Delta v \sim F\Delta t$, with $\Delta t$ characteristic duration of a kick
  - as $\omega = v_g k$, $\Delta t \propto 1/k$ ($\Delta t \propto 1/\sqrt{k}$ if $\omega = 2\sqrt{k}$)
  - during a time $T$, there are $N \sim T/\Delta t \propto k$ kicks ($N \propto \sqrt{k}$ if $\omega = 2\sqrt{k}$)
  - after $N$ kicks, $\langle \Delta v^2 \rangle = N\Delta v^2$
  - hence, $\langle \Delta v^2 \rangle \propto F^2/k$ independent of k ($\langle \Delta v^2 \rangle \propto \sqrt{k}$ if $\omega = 2\sqrt{k}$)



# Influence of the radial decomposition trough $n_{max}$

- Evolution after 500 Myr for different values of $n_{max}$, for $l_{max} = 4$.



# Influence of the orbital decomposition through $l_{max}$

- Evolution after 500 Myr for different values of $l_{max}$, at fixed $n_{max} = 10$.
- When $l_{max} = 0$, the flattening of the density profile is almost imperceptible, but not non-existing. Running the simulation longer yields a similar flattening as with $l_{max} \neq 0$. The condition $l_{max}$ imposes a spherically-symmetric potential at each time step, which smooths out the effects of the perturbations as they are averaged over $\theta$, $\phi$. Energy redistributed through non-isotropic processes.
- For $l_{max} = 8$, the discreteness of the simulation becomes visible.